Mallows Distance as a Mode of Convergence in the Asymptotic Behavior of a Classic Stochastic Process: Recent Results and Statistical Simulations

Eliézer Soares Pereira (eliezer.aleluia@gmail.com)
Instituto Federal de Brasília

Abstract. The present study addresses the characterization of Mallows diatance convergence of the Empirical Process generated by random samples. The theme guides a Scientific Initiation promoted by FAP / DF. The objective is to understand the relationship of the Empirical process with the Gaussian Distribution through Mallows Distance, in particular, order 2 and other orders, providing statistical simulations of these cases. The scientific community will be detailed about the research results and the scope of their motivating problem. Additionally, we will present the elementary results modeling of probability theory by computational implementation. Given a random sample $X_1, X_2, \ldots, X_n$ and its distribution F, by the Law of Great Numbers and the Glivenko-Cantelli Theorem, F can be well estimated by the constructed empirical distribution function F_n from the data. Historically, this fact has given relevance to the study of the stochastic process called the empirical process, in addition, the study of convergences was fostered for the understanding of the asymptotic behavior of data obtained in real situations. Given the empirical process $\sqrt{n}(F_n(x) - F(x)), x \in \mathbb{R}$ associated with $X_1, \ldots, X_n$ of F, we have a relationship with the Gaussian distribution via Central Limit Theorem. Thus, the study of the asymptotic behaviour of empirical process refers to the so called goodness of fit tests, which are routines aimed to try the distributional identity from a given sample. A recent approach to goodness of fit tests is based on the Mallows distance metric. Some Mallows distance applications to Statistics address the relation of this metric and partial sums, according to [3], [1], [2] e [4]. The convergence of empirical process β_n in the metric modality; that is, the asymptotic behaviour of $d_\alpha(\beta_n, Z)$, is a theme of interest because this metric modality is stronger than the classic ones and more widespread lately in statistical simulations, [4] e [3]. The theme itself is also a great student initiation in the modern world of research in Probability and Statistics, as it aggregates a lot of Probability theory concepts and classic facts with two of the latest research trends in production of knowledge in said field, namely: the approach of distributions, which are asymptotic limits to partial sums (called stable distribution), and Mallows distance as a convergence metric.

This work was carried out under the guidance of Prof. Dr. Wembesom Mendes Soares (Instituto Federal de Brasília).

References

[1] BARBOSA, E. G. and DOREA, C. C. Y. **A note on Lindeberg condition for Convergence to stable laws in Mallows distance.** Bernoulli, Vol. 15 (3), pp. 922924, 2009.

[2] DOREA, C. C. Y. and FERREIRA, D. B. **Conditions for Equivalence Between Mallows Distance and Convergence to Stable Laws.** Acta Mathematica Hungarica, Vol. 134(1-2), pp. 1-11, 2012.

[3] JOHNSON, O. and SAMWORTH, R. **Central limit theorem and convergence to stable laws in Mallows distance.** Bernoulli, Vol. 11, pp. 829-845, 2005.

[4] SOARES, Wembesom Mendes. **Convergência em Distância Mallows Ponderada com Aplicações em Somas Parciais e Processos Empíricos.** 2015. 76 f. Tese (Doutorado) - Curso de Matemática, Instituto de Ciências Exatas, Departamento de Matemática, Universidade de Brasília, Brasília, 2015.